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Impact of an ideal fluid jet on a curved wall: the inverse problem

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ABSTRACT. – This paper analyzes an ideal fluid jet impinging a wall. The usual two-dimensional model of jet flow uses an ideal, incompressible, weightless fluid, and maps this flow in a way that reduces it to a problem of complex analysis that cannot be solved analytically. An efficient procedure is presented here for solving the inverse problem numerically in the case of an arbitrary wall shape, i.e. the design of a wall corresponding to a prescribed velocity (or pressure) distribution. In similar studies, as in airfoil design, important constrains have to be applied to the prescribed distribution in order to ensure the existence of a solution. Not only is this not the case here, but also a constraint must be added to impose the uniqueness of the solution. © Elsevier, Paris.

1. Introduction

In the classical two-dimensional theory of jets that Helmholtz and Kirchhoff introduced in 1968, the fluid is ideal, weightless, and incompressible. The flow is assumed to be steady, irrotational, and bounded by walls and free streamlines.

Many different flow problems have been solved with complex analysis by Joukowsky, Villat, Lévi-Civita, and Cisotti, among others. The monographs of Birkhoff and Zarantonello (1957), Jacob (1959), Gurevich (1966), and Milne-Thomson (1968) give good surveys of these.

The main difficulty in these studies stems from the shape of the walls. Only a few geometries with rigid plane wall boundaries can be treated. The reason is that the Schwarz-Christoffel formula is used to map the flow conformally, and this cannot be expressed analytically when the boundary contains four of five corners. Dias *et al.* (1987) nonetheless presented an efficient numerical procedure for computing jets issuing from arbitrary polygonal nozzles. Later, gravity was introduced (see Dias and Vanden-Broeck, 1990).

To consider curved walls, a new approach has been proposed on the basis of previous studies of Helmholtz flows (Hureau *et al.,* 1987). This approach yields a numerical solution for the case of a jet divided by a curved obstacle (Hureau *et al.*, 1996), and for a nozzle with curved wall, even if the emerging fluid is assumed to be heavy (Toison and Hureau, 1997). Recently, Peng and Parker (1997) have studied the case of a jet impinging an uneven wall. They use the Hilbert transform to convert the condition of constant speed on the free streamlines into a relation between the flow angle on the free surface and the wall angle. Then, with the generalized Schwarz-Christoffel transformation technique, they define a system of nonlinear integro-differential equations.

At the same time, we were also considering jet problems, and have written a paper describing a method for solving the problem of impinging free jets numerically (Hureau and Weber, 1998). The solution to this problem is based on the construction of a unique streamline $\mathcal{L}$ in equilibrium separating the two impinging free jets. This is done by a two-part recursive scheme. In the first part, the pressure distribution on $\mathcal{L}$ is determined by

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considering the impact of the first jet on it. This is the direct problem, in a special case of a jet divided by a curved obstacle, and is performed by the Levi-Civita method and the Schwarz-Villat formula. The results are compared with those presented by Peng and Parker. In the second part of the iterative process, we have to modify the geometry of the streamline $\mathcal{L}$ because the pressure distribution must coincide with that of the second jet on the wall. For want of a better term, we refer to this as the "inverse problem".

But the formulation of this *pseudo*-"inverse problem" does not allow us to define the geometry of the impinged wall and then find the free surface of the incoming jet for any given pressure distribution, which would be the inverse problem according to Lighthill (1945). As we have already studied the inverse problem for airfoils, in which we design a physically acceptable airfoil that corresponds to a prescribed surface pressure (Hureau and Legallais, 1996), and since the solution of the inverse problem for a jet impinging on an infinite wall does not exist in the literature, we have analyzed this in depth. Moreover, in the theory of inverse airfoil design, constraints are imposed on the prescribed surface values which, unless satisfied, preclude the existence of a solution. So we will look to see if such constraints still exist in the case of a jet impinging an infinite wall. Another reason for solving an inverse problem of this kind is to investigate the heat transfer problems as mentioned by Peng and Parker. As heat transfers are proportional to velocity, the direct problem can be used to determine the transfers, and the inverse problem to optimize them, by modifying the shape of the wall (see Fig. 6 for an example). Similarly, the optimization of the pressure distribution was at the origin of the inverse problem designing airfoils.

2. Formulation of the problem

Let a jet be divided by an *unknown* curved infinite barrier into two branches bounded by the free streamlines $\mathcal{L}_{\mathrm{AB}}$ and $\mathcal{L}_{\mathrm{AD}}$, as in figure 1. We define the x-axis of the cartesian coordinates as the centreline of the impinging jet, and locate the origin O at its intersection with the wall. We define s to be the arc length onto the unknown wall starting from O, $s \in]-\infty, +\infty[$.

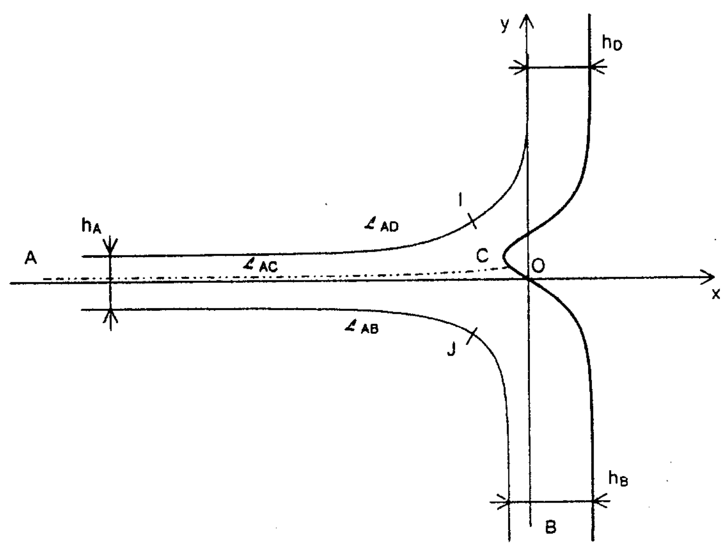

Fig. 1. – Impact of a jet on an infinite wall.

Far from the wall, at point A, the flow is assumed to be uniform, with width h_A and velocity V_∞. Far up and down the wall from the stagnation point C, the flow is also considered to be uniform, and the widths at points B and D are denoted h_B and h_D, respectively.

To solve the inverse problem, i.e. to define the geometry of the impinged wall, we start with a prescribed velocity (or pressure) distribution. We first assume that this distribution $V(s)$ is available, and then analyze the conditions it has to verify.

The boundary conditions are

$$\lim_{z \to A} w(z) = V_\infty \tag{2.1}$$

$$\begin{cases} \mathfrak{Im}\{w(z)dz\} = 0 \\ |w(z)| = V(s) \end{cases} \text{on the unknown wetted wall BCD} \tag{2.2}$$

$$|w(z)| = V_\infty \text{ on } \mathcal{L}_{\text{AB}} \text{ and } \mathcal{L}_{\text{AD}}, \tag{2.3}$$

with f and w being the complex potential and velocity, respectively. The physical plane is described by expressing the complex position z.

3. Theoretical formulation

To solve this inverse problem, we have to determine the geometry of the wetted wall BCD and calculate the two free streamlines $\mathcal{L}_{\text{AB}}$ and $\mathcal{L}_{\text{AD}}$ to define D_z, the flow region in the z-plane. The complex functions f and w are obtained by conformal mapping on an auxiliary plane ζ. As the region of variation D_f in the f-plane is very simple–a strip of width $h_A V_\infty$ (Fig. 2)–, it is very easy to map. But when the wall is curved, the region of variation in the w-plane is unknown, so we cannot map it onto the ζ-plane.

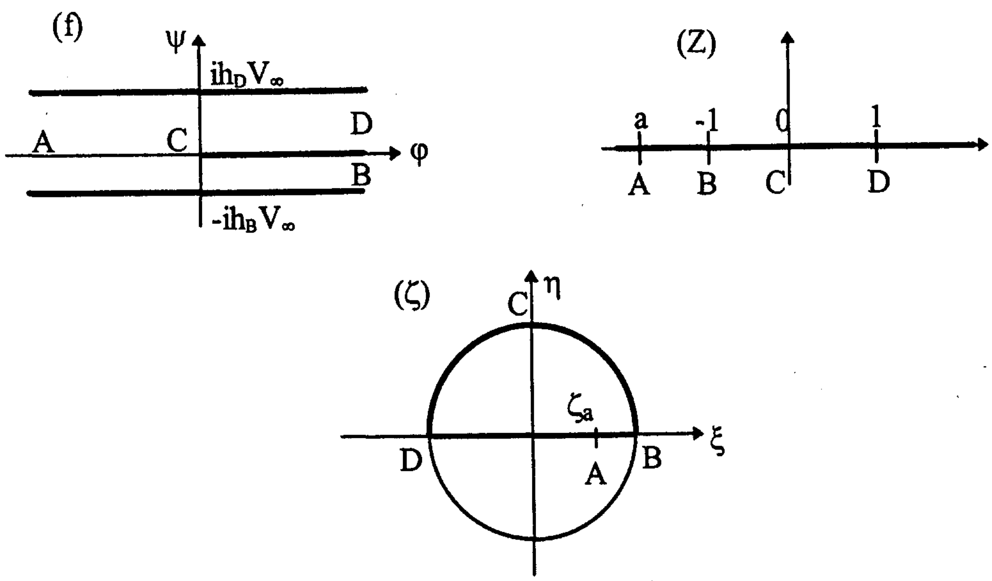

Fig. 2. – Conformal mapping planes.

Let us consider the function Ω :

$$\Omega = -i \, \text{Log} \frac{V_\infty}{w} = \Theta + iT,$$

where Θ is the direction of the velocity $\vec{V}$, and T is given by $|\vec{V}| = V_\infty e^T$. The boundary conditions (2.2) and (2.3) imply that the shape of the wall is given by Θ, and that T is null on the free streamlines $\mathcal{L}_{\mathrm{AB}}$ and $\mathcal{L}_{\mathrm{AD}}$, respectively. On the barrier, T has specified values:

$V_\infty e^{T(s)} = V(s)$ when the prescribed distribution is a velocity distribution

or $1 - e^{2T(s)} = V(s)$ when we have a pressure distribution.

We choose to use the Lévi-Civita method (Lévi-Civita, 1907) in which D_f and D_z are mapped onto the half-unit disk D_ζ so as to map the wetted wall on the half circle and the free streamlines on the diameter. Figure 2 shows the different planes we used. As $\Omega(\zeta)$ tends continuously toward real values on the diameter, the Ω-function can then be continued to the lower half-unit disk, by applying the Schwarz-reflection principle. On the unit circle, $\zeta = e^{i\sigma}$; so we denote $\tau(\sigma) = T(e^{i\sigma})$ and $\theta(\sigma) = \Theta(e^{i\sigma})$. The boundary condition (2.2) enables us to define $\tau(\sigma)$ on the circle, and then the Schwarz-Villat formula gives the values of θ.

Now, we map D_f onto the half-unit disk D_ζ of the ζ-plane.

In conformal mapping, we have to prescribe the position of three points. Points B and D are logical choices, so we have to impose one of the two points A or C. In the *pseudo*-"inverse problem" used to solve impinging jets problems, A is put at the origin of the ζ-plane, so the position of C has to be defined. When the point A is fixed at the origin, we obtain an infinite number of solutions depending on the initial length discretization on the wall. To prevent this indeterminacy we need a reference point on the wall. So we fix the location of the stagnation point C in the ζ-plane, and then have to determine the position of A. Let a denote its position in the Z-plane and ζ_a in the ζ-plane. As the location of the stagnation point C is known in the ζ-plane and in the initial velocity (or pressure) distribution ($V = 0$ or $Cp = 1$), we can then take C as the origin of the arc length and of the cartesian coordinates (in place of O). This will be done subsequently.

The Schwartz-Christoffel formula used to map D_z onto D_f is

$$f(Z) = K\left[\frac{a}{a^2-1}\mathrm{Log}(Z-a) + \frac{1}{2(1-a)}\mathrm{Log}(Z-1) - \frac{1}{2(1+a)}\mathrm{Log}(Z+1)\right] + \text{const.},$$

where K is a constant that has to be determined. Analyzing $f(Z)$ in the vicinity of the infinite points A, B, and D in the f and the Z-planes, we get the following three relations, respectively:

$$\begin{cases} ih_A V_\infty = i\dfrac{Ka}{a^2-1}\pi \\ ih_B V_\infty = -i\dfrac{-K}{2(1+a)}\pi, \\ ih_D V_\infty = -i\dfrac{K}{2(1-a)}\pi \end{cases}$$

and thereby

$$\frac{h_B}{h_D} = \frac{a-1}{a+1}. \tag{3.1}$$

The relation $Z = -\frac{1}{2}\left(\zeta + \frac{1}{\zeta}\right)$ maps D_ζ on D_z, so we can say

$$f(\zeta) = \frac{\mathrm{h_A V_\infty}}{\pi}\mathrm{Log}\left[-\frac{1}{2}\left(\zeta+\frac{1}{\zeta}\right) - a\right] - \frac{h_B V_\infty}{\pi}\mathrm{Log}\left[-\frac{1}{2}\left(\zeta+\frac{1}{\zeta}\right) + 1\right] - \frac{h_D V_\infty}{\pi}\mathrm{Log}\left[-\frac{1}{2}\left(\zeta+\frac{1}{\zeta}\right) - 1\right] + \text{const.} \tag{3.2}$$

Let β be the angle between the tangent at a point on the wall and the x-axis, and ϵ the one-to-one correspondence function between the boundary pieces BCD of D_ζ and D_z.

(3.3)
$$\theta(\sigma) = (\beta \circ \varepsilon)(\sigma) - \begin{cases} \pi \text{ for } \sigma \in [0, \pi/2[\\ 0 \text{ for } \sigma \in]\pi/2, \pi] \end{cases}.$$

Ω exhibits a discontinuity at the stagnation point C, because θ has two values and, moreover, $\tau \to -\infty$ there. To isolate the singular part of Ω, we write $\Omega = \widetilde{\Omega} + \Omega_S$, where $\widetilde{\Omega}$ is a regular function, and Ω_S a function which has the same singularity as Ω. In accordance with the Schwarz-reflection principle, the same discontinuity must occur when $\zeta \to -i$. Here, there is an angle π between the velocities at the stagnation point C. A function satisfying these conditions is usually employed (see Gurevich, 1966, or Jacob, 1959):

$$\Omega_S(\sigma) = \theta_S(\sigma) + i\tau_S(\sigma) = -\frac{\pi}{2} + i\,\mathrm{Log}\left(\frac{e^{i\sigma} - i}{e^{i\sigma} + i}\right),$$

so that

$$\theta_S = \begin{cases} -\dfrac{\pi}{2} \text{ for } \sigma \in [0, \pi/2[\\ +\dfrac{\pi}{2} \text{ for } \sigma \in]\pi/2, \pi] \end{cases} \quad \text{and} \quad \tau_S = \ln\left|\frac{\sin\left(\dfrac{2\sigma - \pi}{4}\right)}{\sin\left(\dfrac{2\sigma + \pi}{4}\right)}\right|.$$

We must now find $\widetilde{\Omega} = \widetilde{\theta} + i\widetilde{\tau}$. This can be done using the Schwarz-Villat formula if we assume that we know $\widetilde{\tau}$. This is true because, if we assume that the ϵ-function can be defined, the imaginary part of $\widetilde{\Omega}$ is known and can be expressed:

(3.4)
$$\widetilde{\tau}(\sigma) = \tau(\sigma) - \ln\left|\frac{\sin\left(\dfrac{2\sigma - \pi}{4}\right)}{\sin\left(\dfrac{2\sigma + \pi}{4}\right)}\right|.$$

Then the Schwarz-Villat formula on the unit circle yields

(3.5)
$$\widetilde{\theta}(\sigma) = \widetilde{\Theta}(\zeta) = \frac{2}{\pi} \lim_{\zeta \to e^{i\sigma}} \Re\mathrm{e}\left\{\int_0^\pi \frac{\zeta \sin(\sigma')}{1 - 2\zeta\cos(\sigma') + \zeta^2}\,\widetilde{\tau}(\sigma')d\sigma'\right\} + \widetilde{\Theta}(0),$$

where the constant $\widetilde{\Theta}(0)$ has to be determined.

To define ϵ, we will express the norm of dz on the wall. As $dz = \frac{df}{V_\infty}\,e^{i\Omega(\zeta)}$, we have

(3.6)
$$dz = \frac{h_A e^{i\Omega(\zeta)}}{\pi}\left[\frac{1}{\left(\zeta + \dfrac{1}{\zeta}\right) + 2a} - \frac{\left(\zeta + \dfrac{1}{\zeta}\right) - \dfrac{2}{a}}{\left(\zeta + \dfrac{1}{\zeta}\right)^2 - 4}\right]\left(1 - \frac{1}{\zeta^2}\right)d\zeta,$$

so with $\zeta = e^{i\sigma}$,

(3.7)
$$\varepsilon(\sigma) = \int_{\pi/2}^{\sigma} |dz| = \frac{h_A(a^2 - 1)}{|a|\pi}\int_{\pi/2}^{\sigma} \frac{1}{e^{\widetilde{\tau}(\sigma')}}\left|\frac{\cos(\sigma')}{\sin(\sigma')[\cos(\sigma') + a]}\right|d\sigma',$$

where the value of a is unknown.

The two values $\widetilde{\Theta}(0)$ and a are undetermined. First, we will examine the unknown $\widetilde{\Theta}(0)$. If we write $\Omega(\zeta) = \Omega'(\zeta) + \widetilde{\Theta}(0)$, where $\Omega'(\zeta)$ is completely defined for a given ζ, we have

$$dz = \left[\frac{df}{V_\infty} e^{i\Omega'(\zeta)}\right] e^{i\widetilde{\Theta}(0)}.$$

So a variation of $\widetilde{\Theta}(0)$ implies a rotation in the ζ-plane. If we are able to draw the stagnation line $\mathcal{L}_{AC}$, its direction at infinity will give us the value of $\widetilde{\Theta}(0)$, with respect to the x-axis as defined, which corresponds to the centreline of the jet.

The determination of ζ_a (or a) is not so immediate. We even notice that, if the value of a is specified, we have, from (3.1) and the conservation of mass ($h_A = h_B + h_D$), the relations

$$\begin{cases} h_B = \dfrac{h_A}{2}\dfrac{a-1}{a} \\ h_D = \dfrac{h_A}{2}\dfrac{a+1}{a} \end{cases}. \tag{3.8}$$

Moreover, all the equations can be solved, a solution can be reached, and we can draw the stagnation line $\mathcal{L}_{AC}$ and the two free streamlines $\mathcal{L}_{AB}$ and $\mathcal{L}_{AD}$. This also gives other values for h_B and h_D in the vicinity of point A. If a is wrong, the value of h_B (or h_D) obtained by equation (3.8) will not coincide with those obtained by the drawing of $\mathcal{L}_{AC}$, $\mathcal{L}_{AB}$, and $\mathcal{L}_{AD}$. An iterative scheme based on the difference Δh between the widths h_B (or h_D) is chosen to determine the value of a.

4. Numerical scheme

The unknowns are the functions $\sigma \to \widetilde{\tau}, \widetilde{\theta}, \varepsilon$, and the value of a. For an initial value a_0, the equations (3.4) and (3.7) supply a functional system:

$$\widetilde{\tau} = f_1(\varepsilon) \qquad (3.4)$$
$$\varepsilon = f_2(\widetilde{\tau}). \qquad (3.7)$$

From an initial function ε_0, we solve by using the following recursive scheme:

$$\begin{cases} \widetilde{\tau}_n = f_1(\varepsilon_{n-1}) \\ \varepsilon_n = (1 - r_2) f_2(\widetilde{\tau}_n) + r_2 \varepsilon_{n-1}. \end{cases}$$

We choose the weighting factor r_2 belonging to [0.5,0.9] and, when the process converges, we stop it by means of a test on the relative error associated with ϵ. To draw the stagnation streamline and the free streamlines $\mathcal{L}_{AB}$ and $\mathcal{L}_{AD}$, we use equation (3.6). For $\mathcal{L}_{AB}$ and $\mathcal{L}_{AD}$, we determine the affix ζ_I and ζ_J corresponding to the points I and J, respectively. We then obtain their location in the z-plane by integrating (3.6) from $\zeta = e^{i\gamma}$ to ζ_I or ζ_J. After this, we calculate $\int_{\zeta_I}^{\zeta\to 1} dz$ and $\int_{\zeta_I}^{\zeta\to 0} dz$ to draw $\mathcal{L}_{AB}$, and $\int_{\zeta_J}^{\zeta\to -1} dz$ and $\int_{\zeta_J}^{\zeta\to 0} dz$ to draw $\mathcal{L}_{AD}$. We thus find $\widetilde{\Theta}(0)$ and evaluate Δh with equation (3.8). The recursive scheme is reiterated for any value a_i, and the whole system is stopped with a test on the relative error associated with the difference Δh. Then $\widetilde{\theta}$ is calculated and the wall is drawn.

The algorithm is presented in figure 3.

5. Existence and uniqueness

We should note that we found no constraint that the prescribed velocity distribution had to satisfy during the solution. In airfoil design (Lighthill, 1945 and Hureau and Legallais, 1996), a solution exists only if certain

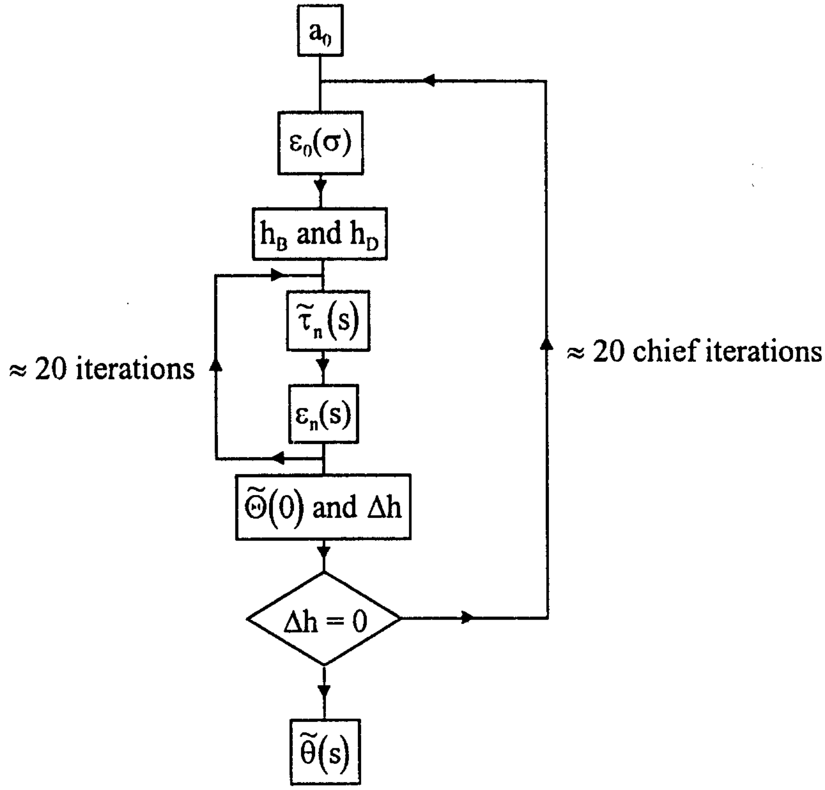

Fig. 3. – Algorithm.

integral constraints are satisfied. Here, there is no integral constraint, but two restrictions still exist: the speed of the outgoing jets downstream must be equal to that of the incoming one upstream, and a stagnation point must exist. This means that we must be able to design a wall from any specified function $\widetilde{\tau}(s)$ that becomes null as the arc length s becomes infinite.

To our knowledge, there is no solution for this problem in the literature. So, to verify our computed results, we use the direct problem (Hureau and Weber, 1998), which is applied to a prescribed wall geometry. The velocity distribution then obtained is the initial one for the inverse problem. We can compute a wall this way, which must be the same as the one used in the direct problem.

An additional constraint is needed to ensure the uniqueness of the solution. We can set the height of the stagnation line, denoted y_{AC}. This line is determined with respect to the jet axis by the equation

$$\mathfrak{Im}\left\{ z_C + \int_{\mathcal{L}_{AC}} dz \right\} = \frac{h_A}{2} - h_D,$$

and with the equation of mass

$$y_{AC} = \mathfrak{Im}\left\{ \int_{\mathcal{L}_{AC}} dz \right\} = h_A \left(\frac{1}{2} - \frac{1}{\dfrac{h_B}{h_D} + 1} \right) - \mathfrak{Im}\{z_C\}.$$

We set this height first because, when we compare our results and the jets computed by the direct problem, we see that the stagnation line geometry is good but expanded. But since the aim of this work is to improve the heat transfer from a fluid to a wall, it is physically more representative to set h_B/h_D. We therefore define the value of a from equation (3.8), and this greatly simplifies the numerical procedure. The 20 chief iterations (Fig. 3) are useless, and the solution is obtained quickly.

6. Results

Generally, the recursive scheme converges in about 20 iterations, corresponding to about one minute of calculation on a PC Pentium 120 MHz.

The first velocity distribution is the one obtained with the impact of a jet of width 2 on an infinite plane inclined at an angle of 20° (Fig. 4 (a)). The direct problem yields $h_B/h_D \approx 0.49029$. Figure 4 (b) shows the initial wall and the corresponding jet, along with the wall and jet computed by the inverse problem with this ratio h_B/h_D ($y_{AC} \approx 0.53866$). The stagnation line is plotted in a broken line. The drawing is exactly the same as the initial barrier. For other values of h_B/h_D (0.80 and 0.30), the computed walls are shown in figure 4 (c) and (d). In these three cases, the velocity distribution on the walls is the same as the initial one.

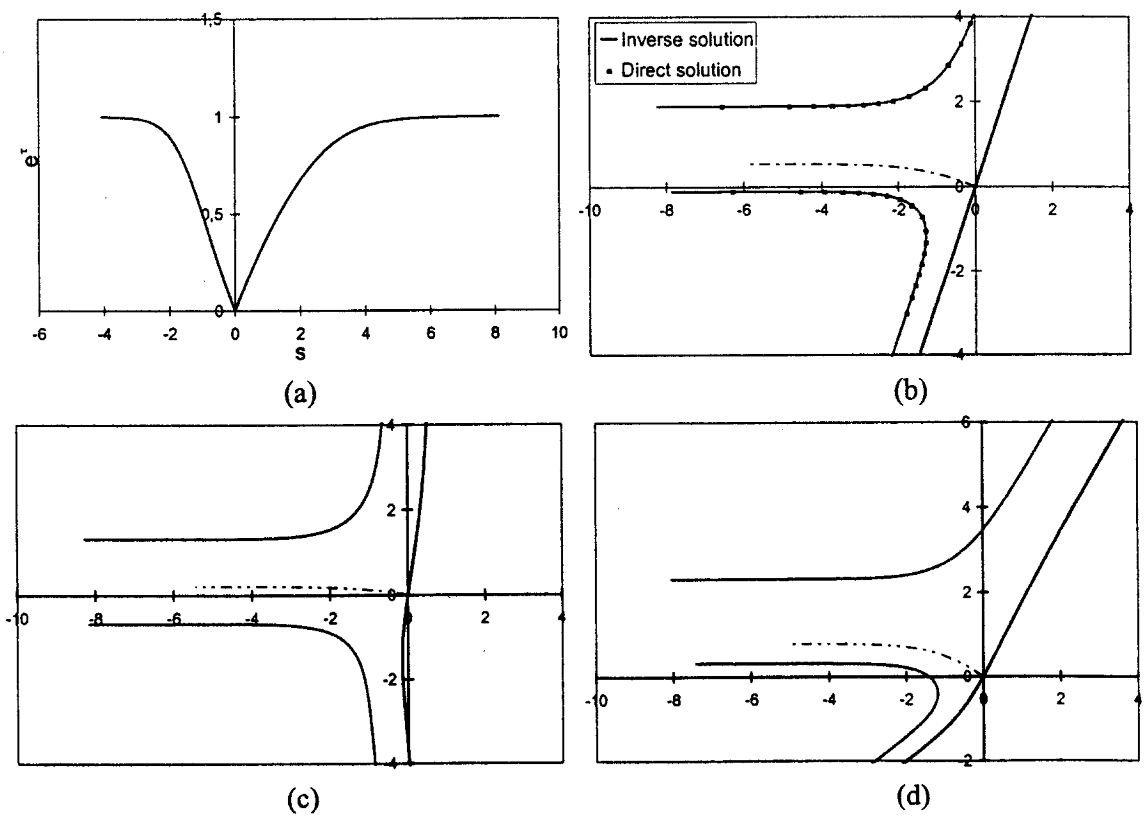

Fig. 4. – (a) Velocity distribution for a jet of width 2 on an infinite plane inclined at an angle of 20°. Computed results for (b) $h_B/h_D = 0.49029$, (c) 0.8, and (d) 0.3.

We can then consider other wall shapes. First, the initial velocity distribution (Fig. 5 (a)) is given by the direct problem for a wall geometry given by $x(y) = -1.2\operatorname{sec}h(y - 0.85)$ with a jet width of 2. This configuration has been studied by Peng and Parker (1997) for the direct problem. We have computed it too (Hureau and Weber, 1998) and find $h_B/h_D \approx 1.37601$. The computed walls are plotted in figure 5 (b) and (c) for $h_B/h_D = 2.10$

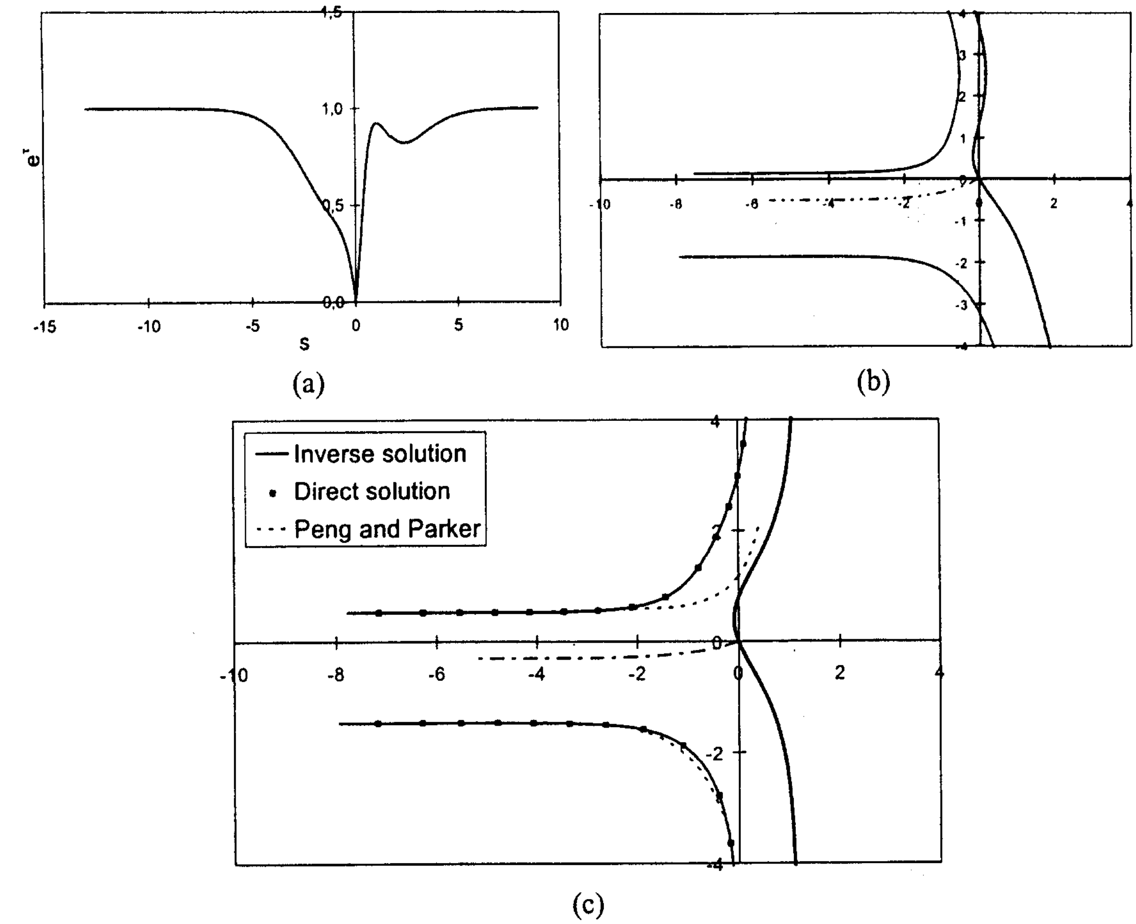

Fig. 5. – (a) Velocity distribution for a jet of width 2 impinging a wall of geometry $x(y) = -1.2\mathrm{sech}(y - 0.85)$. Computed results for (b) $h_B/h_D = 2.10$, and (c) 1.37601.

and $h_B/h_D = 1.37601$, respectively. In this last case, the flow patterns coincide between the direct and the inverse solutions, but not with the one presented by Peng and Parker. We should point out that the conservation of mass does not seem to be verified in the Peng and Parker results.

To get the results of figure 6, we used the direct method to find an initial velocity distribution (Fig. 6 (a)–direct distribution), for a wall geometry defined by three sinusoidal periods with wavelengths of 2, extended to infinity by two plates. The jet width is 1, and its centreline is shifted a quarter-period from the wall's axis of symmetry. The direct problem gives $h_B/h_D \approx 1.05341$. In figure 6 (b) and (c), we have the computed walls for $h_B/h_D = 1.05341$ (to find the corresponding initial wall) and 0.40, respectively.

With this configuration, we then modified the initial pressure distribution a little (Fig. 6 (a)–modified distribution), and computed the corresponding wall for $h_B/h_D = 1.05341$ (Fig. 6 (d)). We noticed a few changes in the shape of the wall. This can be taken as an example of obstacle geometry optimization in heat transfer problems.

In a first approximation, an even discretization can be used on the half-unit circle of the ζ-plane, e.g. with 721 points (one point every 0.5°). As this method requires a major distortion of the boundary in the vicinity of B and D, it is better to discretize the half-unit circle in the vicinity of 0 and π with many more quadrature

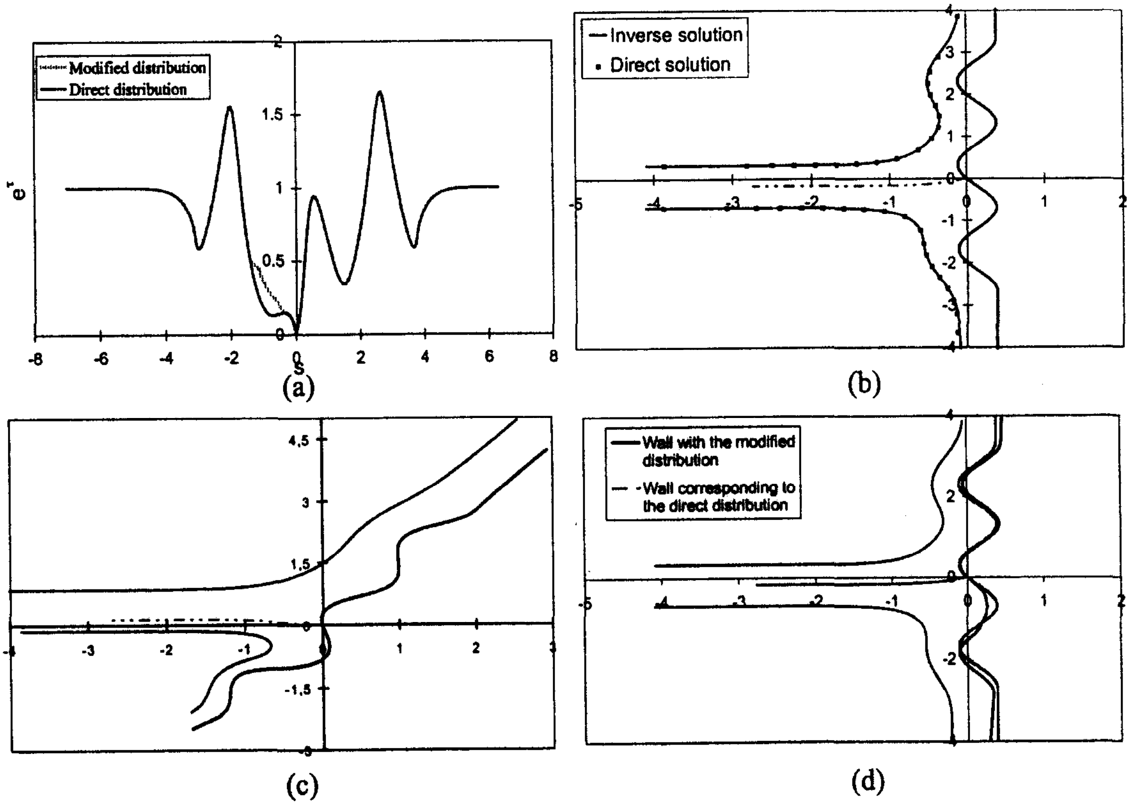

Fig. 6. – (a) Velocity distribution for a jet of width 1 (direct distribution) impinging three sinusoidal periods extended to infinity. Computed results for (b) $h_B/h_D = 1.05341$, and (c) 0.40. (d) Computed results for the modified pressure distribution for $h_B/h_D = 1.05341$.

points (100 points in the first and last 5°). But even with this discretization, the infinite points B and D are at best represented by points with arc length s no greater than about four or five jet widths in norm. Infinity is never reached, and this limits the testing possibilities.

As $\tilde{\tau}$ may be arbitrary, and if we add that a stagnation point must exist and Cp must be zero at infinity, we create a function $e^{\tau(s)}$ which is defined even when s becomes very large. The functions used are:

$$\begin{cases} e^{\tau(s)} = \sqrt{1 - e^{-s^2}} & \text{for } s < 0 \\ e^{\tau(s)} = \sqrt{1 - sec\,h(s)} & \text{for } s > 0 \end{cases},$$

and the results are plotted in figure 7 (a). Figures 7 (b), (c), and (d) show the shapes of the walls that we computed with $h_B/h_D = 1.2$, 0.5, and 0.05, respectively. The jet width is 2. The third case is an extreme configuration because the ratio between the two outgoing jet widths is large. The enlargement in the vicinity of the stagnation point in this case is plotted in figure 7 (e). The crossing of flow has nonphysical meaning, and occurs because nothing in our formulation prevents this from happening. This phenomenon is also seen for impinging jets (Hureau and Weber, 1998), jets issuing from nozzles (Dias *et al.,* 1987), and flows over polygonal obstacles (Elcrat and Trefethen, 1986). To determine the flow more realistically, the mathematical model has to be revised.

Note here that symmetrical walls cannot be dealt with in this formulation of the problem: the value ζ_a is automatically set at 0 and $\widetilde{\Theta}(0) = 0$. This creates a numerical problem ($|a| = +\infty$). The equations (3.2), (3.6),

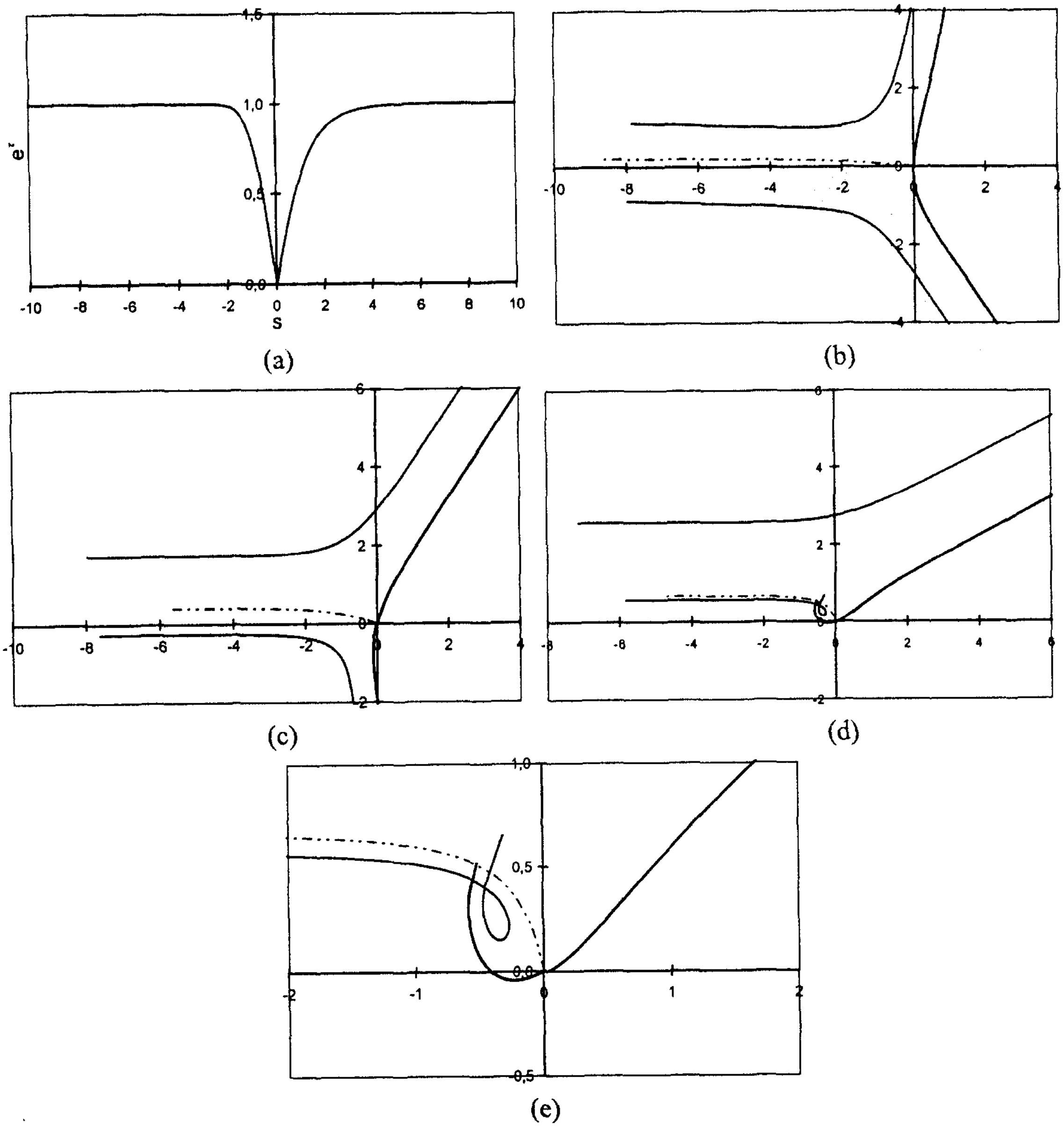

Fig. 7. – (a) Analytical velocity distribution for a jet of width 2: $e^{\tau(s)} = \sqrt{1-e^{-s^2}}$ for $s < 0$ and $e^{\tau(s)} = \sqrt{1-\mathrm{sech}(s)}$ for $s > 0$. Computed results for (b) $h_B/h_D = 1.2$, (c) 0.5 and (d) 0.05. (e) Enlargement.

and (3.7) have to be rewritten and become similar to the corresponding ones of the direct problem:

$$f(\zeta) = -\frac{h_B V_\infty}{\pi} \operatorname{Log}\left[-\frac{1}{2}\left(\zeta+\frac{1}{\zeta}\right)+1\right] - \frac{h_D V_\infty}{\pi} \operatorname{Log}\left[-\frac{1}{2}\left(\zeta+\frac{1}{\zeta}\right)-1\right] + \text{const.},$$

$$dz = \frac{e^{i\Omega(\zeta)}}{\pi}\left[\frac{h_B}{2-\left(\zeta+\frac{1}{\zeta}\right)} - \frac{h_D}{2+\left(\zeta+\frac{1}{\zeta}\right)}\right]\left(1-\frac{1}{\zeta^2}\right) d\zeta,$$

and

$$\varepsilon(\sigma) = \frac{2h_A}{\pi} \int_{\pi/2}^{\sigma} \frac{1}{e^{\tilde{\tau}(\sigma')}} \frac{\sin^2\left(\frac{2\sigma'+\pi}{4}\right)}{|\sin(\sigma')|} d\sigma'$$

where $h_B = h_D = \frac{h_A}{2}$. The solution of the problem is the same as for asymmetrical walls.

7. Concluding remarks

From the results that we present, we believe that the problem of designing a wall to obtain a prescribed velocity distribution when impinged by a jet is solved. As this method requires a major distortion of the boundary in the vicinity of B and D, there are some difficulties calculating Cp or s for points at a distance of more than about five jet widths from the stagnation point. This is the usual limitation with this kind of method: the crowding, well known in numerical conformal mapping. This occurs for both the direct and the inverse problems.

There is another restriction concerning the computation of the free streamlines that bound the jet. It is not possible to draw these streamlines correctly as we approach B and D because the Schwarz-Villat formula cannot be used to compute Ω accurately, and consequently dz, near the extremes on the diameter of the unit circle. The difference between the widths of the outgoing jets on the plots presented by Peng and Parker may be due to similar reasons. Nevertheless, their drawings are not very large.

We should note that the inverse problem of a jet impinging a wall contains only two constraints on the prescribed surface values: a stagnation point must exist and the velocity at infinity must be equal to V_∞. Moreover, we have to set an additional condition to obtain a unique numerical solution. We have not proved here that the solution is unique, but only verified it by computing the direct problem. In ordinary airfoil design problems, for Lighthill Cp(s) is given on the mapped circle, three integral constraints are required on the prescribed velocity distribution which, unless satisfied, preclude the existence of a solution. For us, when the velocity distribution is given on the profile chord, only one integral constraint is required.

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